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TRANSIENT RESPONSE OF TWO FLUID-COUPLED CYLINDRICAL ELASTIC SHE--ETC(U)

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Transient Response of Two Fluid-Coupled Cylindrical Elastic Shells to an Incident Pressure Pulse

LEVEL II

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Structures Group
Ocean Technology Division

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equations in the time domain. A sample calculation of shell response was performed and results are compared to the case in which the outer shell is absent. It is found that the primary effects of a thin outer shell could be estimated by simple asymptotic formulae.

CONTENTS

TRANSIENT RESPONSE OF TWO FLUID-COUPLED CYLINDRICAL ELASTIC SHELLS TO AN INCIDENT PRESSURE PULSE

INTRODUCTION

In order to gain physical insight in the response as well as to provide a data base for the development of general numerical methods for predicting the underwater explosion response of fluid coupled shell systems such as the double hull section of a submarine, the transient response of systems of fluid coupled elastic shells of simple configurations to the excitations of incident pressure pulse are being analytically investigated. The case of two concentric spherical elastic shells lends itself to the classical treatment of the separation-of-variable and integral transform techniques and accurate solutions have been obtained by a successive integration scheme [1]. It was found that a thin outer shell tends to be transparent to the incident pulse.

In the present report, a solution method and results for the case of two concentric cylindrical elastic shells of infinite length are presented. This is also one of the cases in which it is possible to apply the separation-of-variable and Laplace transform techniques to simultaneously solve the wave equations governing the wave motions in the fluids and the equations of motion of the elastic shells. The corresponding problem for a single cylindrical shell has been widely studied and the results have been reviewed and summarized in many articles, e.g., references [2,3]. Here, it was more convenient to obtain the inverse Laplace transform for the transient solution indirectly using a integral equation method [4] or a differential-

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integral equation method [5]. It is also possible to formulate the present double-cylindrical-shell problem such that the same integral equation method can be used to calculate the transient shell response.

Results are obtained for a system of two steel shells submerged in water. Similar to the spherical case [1], it is also found that a thin outer shell does not change significantly the basic response feature of the inner shell and that its primary effects are to reduce the axi-synmetric (breathing) and translational motions of the inner shell. These lower mode effects can be estimated by simple formulae.

DESCRIPTION OF THE PROBLEM

Figure 1 sketches the submerged fluid-coupled cylindrical shell system and the incident plane pressure wave. The fluid surrounding the outer shell and that between the two shells are considered to be ideal compressible fluids in linear wave motions and can be characterized by their unperturbed mass densities and sound speeds (ρ^e, c^e) and (ρ, c) respectively. The shells are initially concentric. In this study, the strength of the incident wave is sufficiently weak such that the shell deflections are elastic and small and deviation from the concentricity remains negligible for the time duration of interest. The middle surface radii, thickness, mass densities, Young's moduli and poisson's ratios of the outer and inner shells are (a^e, h^e, ρ_s^e, E^e, ν^e) and (a, h, ρ_s, E, ν), respectively. The z-coordinate of the cylindrical coordinate system (r, θ, z) coincides with the axis of the shells in its unperturbed position. The incident plane wave front is also parallel to the z-axis and therefore all response entities do not vary with respect to z.

The deflections of the inner shell in the r- and θ- direction, normalized with respect to the outer shell radius a^e, are denoted by w and v respectively and those of the outer shell by w^e and v^e respectively. The deflections in the z-direction do not enter into the present two-dimensional system of equations of motion. The total pressure field exterior to the outer shell is denoted by $p^e(r, \theta, t)$ and that between the shells by $p\ (r, \theta, t)$ where t designates time. The following dimensionless parameters are also used in the mathematical formulation:

$$
\begin{aligned}
&R = r/a^e, \quad T = c^e t/a^e, \quad \zeta = a/a^e, \quad \eta = c_r\,(1-\zeta),\\
&c_r = c^e/c, \quad \rho_r = \rho/\rho^e, \quad M = \rho^e a/(\rho_s h), \quad M^e = \rho^e a^e/(\rho_s^e h^e),\\
&P = p/[\rho^e (c^e)^2], \quad P^e = p^e/[\rho^e (c^e)^2], \quad I = \frac{1}{12}\,(h/a)^2, \quad I^e = \frac{1}{12}\,(h^e/a^e)^2,\\
&C^2 = E/[\rho_s (1-\nu^2)(c^e)^2], \quad C_e^2 = E^e \Big/ \left\{\rho_s^e\,[1-(\nu^e)^2](c^e)^2\right\},\\
&\alpha_0 = \alpha_0^e = \mu_0 = \mu_0^e = 0, \quad \tau_0 = C^2, \quad \tau_0^e = C_e^2\,,\\
&\alpha_n = n^2\,C^2(1+I), \qquad \alpha_n^e = n^2\,C_e^2(1+I^e)\\
&\tau_n = C^2(1+n^2)(1+n^2 I), \qquad \mu_n = I\,C^4\,n^2\,(1-n^2)^2\,,\\
&\tau_n^e = C_e^2\,(1+n^2)(1+n^2 I^e), \qquad \mu_n^e = I^e C_e^4\,n^2(1-n^2)^2\,,\\
&n = 1, 2, 3, \ldots
\end{aligned}
\tag{1}
$$

P^e and P satisfy the wave equations

$$\nabla^2 pe = \frac{\partial^2 pe}{\partial\,T^2} \tag{2}$$

and

$$\nabla^2 P = c_r^2\,\frac{\partial^2 P}{\partial\,T^2} \tag{3}$$

respectively, where ∇^2 is the Laplacian operator. The boundary conditions of the problem are that P^e satisfies the radiation condition at far field and that

$$\frac{\partial P}{\partial R} = \frac{\partial P^e}{\partial R} = -\frac{\partial^2 w^e}{\partial T^2} \quad \text{at} \quad R = 1 \tag{4}$$

and

$$\frac{\partial P}{\partial R} = -\rho_r \frac{\partial^2 w}{\partial T^2} \quad \text{at} \quad R = \zeta \, . \tag{5}$$

All quantities except the incident press[illegible] field have quiescent initial conditions.

A Laplace transform pair is defined as

$$\begin{aligned} \overline{w}\,(\theta,s) &= \int_0^\infty w\,(\theta,T)\, e^{-sT}\, d_T \\ w\,(\theta,T) &= \int_{\gamma-i\infty}^{\gamma+i\infty} \overline{w}\,(\theta,s)\, e^{sT} ds \end{aligned} \tag{6}$$

where γ lies to the right of all singularities of $\overline{w}$ in the complex s-plane and $i = (-1)^{1/2}$.

Due to the circular geomtry of the problem, the solutions can be expanded in Fourier series as the following:

$$\begin{aligned} P(R,\theta,T) &= \sum_{n=0}^{\infty} P_n(R,T)\cos n\,\theta \\ P^e(R,\theta,T) &= \sum_{n=\infty}^{\infty} P_n^e\,(R,T)\cos n\,\theta \\ w\,(\theta,T) &= \sum_{n=0}^{\infty} w_n(T)\cos n\,\theta \\ v\,(\theta,T) &= \sum_{n=1}^{\infty} v_n\,(T)\sin n\,\theta \\ w^e\,(\theta,T) &= \sum_{n=0}^{\infty} w_n^e\,(T)\cos n\,\theta \end{aligned} \tag{7}$$

$$v^e(\theta,T) = \sum_{n=1}^{\infty} v_n^e(T)\sin n\theta$$

This type of expansion is not suitable for the pressures and shell accelerations at early time [3,4]. Otherwise, it greatly facilitates the calculation of the responses of the shells.

For the solution scheme to be used here, any linear elastic theory of the cylindrical shell is applicable. In order to juxtapose solutions to those for a single shell previously obtained in Ref. [4], the version of shell equations of motion used therein is also used here. In the Laplace transform domain, they are

$$\bar{w} = \frac{-M\zeta(\zeta^2 s + \alpha_n)\,\bar{P}_n(\zeta,s)}{\zeta^4 s^4 + \tau_n \zeta^2 s^2 + \mu_n}$$

$$\bar{w}_n^e = \frac{-M^e(s^2 + \alpha_n^e)\,[\bar{P}_n^e(1,s) - \bar{P}_n(1,s]}{s^4 + \tau_n^e s^2 + \mu_n^e} \tag{8}$$

$n = 0, 1, 2, \ldots$

and

$$(\zeta^2 s^2 + \alpha_n)\,\bar{v}_n = -n\,C^2\,(1+n^2 I)\,\bar{w}_n$$

$$(s^2 + \alpha_n^e)\,\bar{v}_n^e = -n\,C_e^2\,(1+n^2 I^e)\,\bar{w}_n^e \tag{9}$$

$n = 1, 2, 3, \ldots$

The total pressure field exterior to the outer shell consists of the pressure due to the incident wave and those due to scattering and radiation by the outer shell. An arbitrary incident plane pressure wave impinging the outer shell at ($\theta=0$, $R=1$) at $T=0$ can be expressed by the following series [4].

$$\overline{P}^{i}\ (R,\theta,s) = f(s)e^{-s}\sum_{n=0}^{\infty} \varepsilon_n I_n(sR)\cos n\,\theta \tag{10}$$

where

$f(s)$ is the Laplace transform of the temporal characteristics of the incident wave, $I_n(sR)$ is the modified Bessel function of the first kind and

$$\begin{aligned}\varepsilon_n &= 1 \text{ for } n = 0\\ &= 2 \text{ for } n > 0\ .\end{aligned} \tag{11}$$

SOLUTIONS IN THE LAPLACE TRANSFORM DOMAIN

It can be readily shown that the solutions to the system of Eqs. (2) through (11) are:

$$\overline{P}^{e}_{n}\ (R,s) = f(s)\ e^{-s}\varepsilon_n \left[I_n(sR) - \frac{I_n'(s)\ K_n(sR)}{K_n'(s)} \right] - \frac{s\overline{w}^{e}_{n}}{K_n'(s)} K_n(sR) \tag{12}$$

$$\begin{aligned}\overline{P}_n(R,s) = {} & \frac{s\overline{w}^{e}_{n}\ K_n'(c_r\zeta s) - s\overline{w}_n\ K_n'(c_r s)}{A_n\ (s)} \left(\frac{\rho c}{\rho^e c^e}\right) I_n(c_r sR)\\ & + \frac{s\overline{w}_n I_n'\ (c_r s) - s\overline{w}^{e}_{n}\ I_n'(c_r\zeta s)}{A_n(s)} \left(\frac{\rho c}{\rho^e c^e}\right) K_n\ (c_r s\ R)\end{aligned} \tag{13}$$

$$\begin{aligned}\overline{w}^{e}_{n} = {} & M^e(s^2+\alpha^e_n)\ [(\zeta^4 s^4+\tau_n\zeta^2 s^2+\mu_n)\ A_n(s)(\rho^e c^e)/(\rho c)\\ & - M\zeta s(\zeta^2 s^2+\alpha_n)B_n(s)]\ \frac{f(s)e^{-s}\varepsilon_n}{\Delta_n\ (s)}\end{aligned} \tag{14}$$

and

$$\overline{w}_n = MM^e(\zeta^2 s^2+\alpha_n)(s^2+\alpha^e_n)\ \frac{f(s)e^{-s}\varepsilon_n}{c_r\ \Delta_n(s)} \tag{15}$$

where $K_n(s)$ is the modified Bessel function of the second kind, the prime denotes differentiation of the Bessel functions with respect to their arguments,

$$\Delta_n(s)=s(\zeta^4s^4+\tau_n\zeta^2s^2+\mu_n)[K_n'(s)(s^4+\tau_n^e s^2+\mu_n^e)-sK_n(s)M^e(s^2+\alpha_n^e)]A_n(s)(\rho^e c^e)/(\rho c)$$
$$-s^2K_n'(s)[M^e(s^2+\alpha_n^e)(\zeta^4s^4+\tau_n\zeta^2s^2+\mu_n)G_n(s)+M\zeta(\zeta^2s^2+\alpha_n)(s^4+\tau_n^e s^2+\mu_n^e)B_n(s)]$$
$$+MM^e\zeta s^3(\zeta^2s^2+\alpha_n)(s^2+\alpha_n^e)[K_n(s)B_n(s)+K_n'(s)L_n(s)(\rho c)/(\rho^e c^e)] \qquad (16)$$

$$A_n(s)=I_n'(c_r\zeta s)K_n'(c_r s)-I_n'(c_r s)K_n'(c_r\zeta s) \qquad (17)$$

$$B_n(s)=I_n(c_r\zeta s)K_n'(c_r s)-I_n'(c_r s)K_n(c_r\zeta s) \qquad (18)$$

$$G_n(s)=I_n(c_r s)K_n'(c_r\zeta s)-I_n'(c_r\zeta s)K_n(c_r s) \qquad (19)$$

and

$$L_n(s)=I_n(c_r s)K_n(c_r\zeta s)-I_n(c_r\zeta s)K_n(c_r s) \ . \qquad (20)$$

$\bar{v}_n$ and $\bar{v}_n^e$ can be found from Eq. (9). Strains and stresses of the shells can be computed using appropriate strain-deflection and stress-strain relationships.

With use of the Tauber's theorem of Laplace transform [16], some of the asymptotic behaviors of the shell responses at late time can readily be revealed from Eqs. (14) and (15). Specifically, for the case where the incident wave P^i is a unit step wave, i.e., $f(s) = 1/s$,

$$\underset{T\to\infty}{w_0^e(T)} = \frac{-M^e[c_r^2C^2(1-\zeta^2)+2\rho_r\zeta^2M]}{c_r^2C^2C_e^2(1-\zeta^2)+2\rho_rM^eC^2+2\rho_r\zeta^2MC_e^2} \qquad (21)$$

$$\lim_{T\to\infty} w_0(T) = \frac{-2\rho_r \zeta M M^e}{c_r^2 C^2 C_e^2 (1-\zeta^2) + 2\rho_r M^e C^2 + 2\rho_r \zeta^2 M C_e^2} \tag{22}$$

$$\lim_{T\to\infty} \dot{w}_1^e(T) = \frac{-4M^e[(1-\zeta^2)+0.5\rho_r M(1+\zeta^2)]}{2(1-\zeta^2)(2+M^e)+2\rho_r(1+\zeta^2)(M+M^e)+\rho_r M M^e[1+\zeta^2+c_r(1-\zeta^2)]} \tag{23}$$

and

$$\lim_{T\to\infty} \dot{w}_1(T) = \frac{-4\rho_r M M^e}{2(1-\zeta^2)(2+M^e)+2\rho_r(1+\zeta^2)(M+M^e)+\rho_r M M^e[1+\zeta^2+c_r C1-\zeta^2)]} \tag{24}$$

where the dot denotes differentiation with respect to T.

Eqs. (21) and (22) give the late time shell deflections long after the incident wave has engulfed the outer shell and they can also be obtained by static analyses. Eqs. (23) and (24) are expressions for the late time translational velocities of the shells in the direction of propagation of the incident wave. It can be shown that the only condition under which $\dot{w}_1(\infty) = \dot{w}_1^e(\infty)$ is $\rho_r M=2$, i.e., when the inner shell is neutrally buoyant in the interior fluid. Otherwise, the ratio between them depends only on the buoyancy of the inner shell and the radius ratio ζ. In Fig. 2, it can be seen that a positively buoyant inner shell ($\rho_r M > 2$) translates faster and a negatively buoyant one ($\rho_r M < 2$) translates slower than the outer shell at late time.

Formulae (21) through (24) can be used to facilitate parametric studies of the effects of various shell and fluid properties on the axisymmetric and translational responses of the shells. They also provide asymptotic checks for the numberical calculations.

THE INVERSION OF THE TRANSFORMED SOLUTIONS

The transformed solutions in Eqs. (12) through (15) are highly complicated functions of s. To find their inverse transforms, it is more convenient to rearrange them such that the solutions in the time domain can be obtained by numerically solving a Volterra type of integral equation the kernel of which can be accurately calculated. This procedure has been demonstrated to be quite effective in previous studies of the single cylindrical shell problem [4,5].

To proceed, Eq. (15) is rearranged to assume the following form:

$$\bar{w}_n(s) - \bar{w}_n(s) \Bigg\{ \frac{M^e s(s^2+\zeta_n^e)}{(s^4+\tau_n^e s^2+\mu_n^e)} \frac{K_n(s)}{K_n'(s)} \left[1 - \left(\frac{\rho c}{\rho^e c^e}\right) \frac{M\zeta s(\zeta^2 s^2+\alpha_n)}{(\zeta^4 s^4+\tau_n \zeta^2 s^2+\mu_n)} \frac{B_n(s)}{A_n(s)} \right]$$

$$+ \left(\frac{\rho c}{\rho^e c^e}\right) \frac{M\zeta s(\zeta^2 s^2+\alpha_n)}{(\zeta^4 s^4+\tau_n \zeta^2 s^2+\mu_n)} \frac{B_n(s)}{A_n(s)} + \left(\frac{\rho c}{\rho^e c^e}\right) \frac{M^e s(s^2+\alpha_n)}{(s^4+\tau_n^e s^2+\mu_n^e)} \frac{G_n(s)}{A_n(s)}$$

$$- \left(\frac{\rho c}{\rho^e c^e}\right)^2 \frac{M\zeta s(\zeta^2 s^2+\alpha_n)}{(\zeta^4 s^4+\tau_n \zeta^2 s^2+\mu_n)} \frac{M^e s(s^2+\alpha_n^e)}{(s^4+\tau_n^e s^2+\mu_n^e)} \frac{L_n(s)}{A_n(s)} \Bigg\}$$

$$= \frac{\rho_r M(\zeta^2 s^2+\alpha_n)}{(\zeta^4 s^4+\tau_n \zeta^2 s^2+\mu_n)} \frac{M^e(s^2+\alpha_n^e)}{(s^4+\tau_n^e s^2+\mu_n^e)} \frac{1}{c_r^2 A_n(s)} \left[\frac{f(s) e^{-s} \varepsilon_n}{s K_n'(s)} \right]. \tag{25}$$

The Nielsen's function,

$$W_n(1,T) = \frac{1}{2\pi i} \int_{\gamma-i\infty}^{\gamma+i\infty} \left[\frac{K_n(s)}{K_n'(s)} + 1 \right] e^{sT} ds \quad , \tag{26}$$

has been accurately computed and tabulated by Nielsen [7]. The resultant of the incident and scattering transient pressure acting on a rigid and motionless circular cylinder impinged upon by an incident plane wave,

$$p_n^{rig}(T) = \frac{1}{2\pi i}\int_{\gamma-i\infty}^{\gamma+i\infty} \frac{f(s)e^{-s}\varepsilon_n}{sK_n'(s)}\, e^{sT}ds \tag{27}$$

has also been accurately computed with the aid of the Nielsen's function [4].

In the following inverse Laplace transform integrals,

$$\Gamma_n^a(T) = \frac{1}{2\pi i}\int_{\gamma-i\infty}^{\gamma+i\infty} \frac{M^e s(s^2+\alpha_n^e)\, e^{sT}}{s^4+\tau_n^e s^2+\mu_n}\, ds\ , \tag{28}$$

$$\Gamma_n^b(T) = \frac{1}{2\pi i}\int_{\gamma-i\infty}^{\gamma+i\infty} \left(\frac{\rho c}{\rho^e c^e}\right) \frac{M\zeta s(\zeta^2 s^2+\alpha_n)}{(\zeta^4 s^4+\tau_n\zeta^2 s^2+\mu_n)} \frac{B_n(s)}{A_n(s)}\, e^{sT}ds\ , \tag{29}$$

$$\Gamma_n^c(T) = \frac{1}{2\pi i}\int_{\gamma-i\infty}^{\gamma+i\infty} \left(\frac{\rho c}{\rho^e c^e}\right) \frac{M^e s(s^2+\alpha_n^e)}{(s^4+\tau_n^e s^2+\mu_n^e)} \frac{G_n(s)}{A_n(s)}\, e^{sT}ds\ , \tag{30}$$

$$\Gamma_n^d(T) + \frac{1}{2\pi i}\int_{\gamma-i\infty}^{\gamma+i\infty} \left(\frac{\rho c}{\rho^e c^e}\right)^2 \frac{M\zeta s(\zeta^2 s^2+\alpha_n)}{(\zeta^4 s^4+\tau_n\zeta^2 s^2+\mu_n)} \frac{M^e s(s^2+\alpha_n^e)}{(s^4+\tau_n^e s^2+\mu_n^e)} \frac{L_n(s)}{A_n(S)} e^{sT}ds, \tag{31}$$

and

$$\Gamma_n^e(T) = \frac{1}{2\pi i}\int_{\gamma-i\infty}^{\gamma+i\infty} \frac{\rho_r M(\zeta^2 s^2+\alpha_n)}{(\zeta^4 s^4+\tau_n\zeta^2 s^2+\mu_n)} \frac{M^e(s^2+\alpha_n^e)}{(s^4+\tau_n^e s^2+\mu_n^e)} \frac{e^{sT}}{c_r^2 A_n(s)}\, ds, \tag{32}$$

the integrands are single-valued and analytic in the complex s-plane since it can be shown that $A_n(s)$, $B_n(s)$, $G_n(s)$ and $L_n(s)$ defined in Eqs. (17) through (20) are single-valued and analytic. Their poles are determined by the zeros of $A_n(s)$ and the 4th order polynomials from the shell equations of motion. By the relationships between the Bessel functions J_n and Y_n and the modified Bessel functions I_n and K_n,

$$\frac{2}{\pi} A_n(\pm is) = J_n'(c_r\zeta s)Y_n'(c_r s) - J_n'(c_r s)Y_n'(c_r\zeta s) \tag{33}$$

where the cross product frequently occurs in applied problems involving an annular region and therefore its zeros have been studied and calculated by many authors. Ref. [8] is one of the many publications presenting tabulated tables. The zeros of this cross product are infinite in number and are all real and simple, hence $A_n(s)$ has infinite number of conjugated pairs of imaginary zeros.

The integrals in Eqs. (28) through (32) can now be accurately evaluated by the customary method of residues and they are sums of sine of cosine functions of T. Thereupon, the inverse Laplace transform of Eq. (25) is a Volterra integral equation of the second kind,

$$w_n(T) - \int_0^T S_n(T-\tau)w_n(\tau)d\tau = F_n(T) \tag{34}$$

where

$$S_n(T) = \Lambda_n(T) - \Psi_n(T) + \Gamma_n^b(T) + \Gamma_n^c(T) - \Gamma_n^d(T), \tag{35}$$

$$F_n(T) = \int_0^T \Gamma_n^e(T-\tau)\, P_n^{rig}(\tau)d\tau, \tag{36}$$

$$\Lambda_n(T) = \int_0^T \Gamma_n^a(T-\tau)\, W_n(1,\tau)d\tau - \Gamma_n^a(T), \tag{37}$$

and

$$\Psi_n(T) = \int_0^T \Lambda_n(T-\tau)\, \Gamma_n^b(\tau)d\tau . \tag{38}$$

$F_n(T)$ and $S_n(T)$ are continous and well behaved functions of T. The numercial methods for solving Eq. (34) have been extensively studied and well developed [9]. The solution for $w_n^e(T)$ can be likewise calculated and it is also evident form Eq. (9) that $v_n(T)$ and $v_n^e(T)$ can be obtained by simple convolutions.

RESULTS AND DISCUSSIONS

The integrals in Eqs. (36) through (38) are evaluated numerically by the trapexoidal rule. Eq. (34) is solved using a simple linear multistep method in which the integral is also computed by the trapezoidal rule. Results are obtained for a case in which both shells are made of steel and both fluids are water. The material properties and dimensions involved are such that

$$
\begin{aligned}
& c_r = \rho_r = 1, \; c^2 = c_e^2 = 12.58121 \\
& M^e = 22.09450 \qquad M = 4.41890 \\
& h^e/a^e = 0.00581 \qquad h/a = 0.02905 \\
& \zeta = 0.8 \qquad \eta = 0.2
\end{aligned}
\tag{39}
$$

The incidence is a step wave with $f(s) = 1/s$. For all integrations, a time step $\Delta T = 0.0125$ is used.

In all subsequent figures, the present solutions are plotted in dotted lines and juxtaposed to those previously obtained in Ref. [4] and plotted in solid lines for the case wherein the outer shell is absent.

Figure 3 shows the results for the w_o's, $\dot{w}_1$'s and w_2's. It can be seen that the numerical results for the w_0's and $\dot{w}_1$'s correctly approach their asymptotic values as calculated by Eqs (22) and (24) respectively. At early time, the presence of the outer shell seems to have little effect.

Physically, the motion of the outer shell starts at T = 0 while that of the inner shell at $T = \eta = 0.2$. This is also evident from the mathematical properties of Eqs. (17), (32) and (36). Their large time translational velocities from Eqs. (23) and (24) are respectively $\dot{w}_1^e(\infty) = -1.17516$ and $\dot{w}_1(\infty) = -1.30361$. If the magnitude of the incident pressure is such that $p^i/[\rho^e(c^e)^2] = 1 \times 10^{-3}$, it can be estimated that at T=10 the relative displacement between the axes of the two shells is still one order of magnitude smaller than the thickness of the inner shell. Therefore, for such an instance, the deviation from concentricity of the outer and inner shells would be undoubtedly negligible.

The present w_2 differs slightly from that of the single shell case and the presence of the outer shell seems to have reduced the rate of decreasing of w_2 at large time from its peak value.

The w_n's for higher n's are plotted in Figures 4 and 5. For $0 \leq T \leq 2$, the present w_3 through w_7 are almost the same as those for the case wherein the outer shell is absent. Similar to the single-shell case, these w_n's have markedly different oscillatory characteristics for T > 2 and their oscillation amplitudes are decreased and the periods increased by the presence of the outer shell. Since the magnitudes of these w_n's are much smaller than those of the lower n terms to begin with, the differences caused by the outer shell would be quite insignicant for the calculation of $w(\theta,T)$. It can also be observed from Figures 3, 4 and 5 that w_n converges quite fast and it would suffice to use only eight terms in the series of Eq. (7) to calculate the transient displacements, velocities and strains of the shells for all values of θ and T. To calculate the early time acceleration and pressure, it is a customary practice to transform Eq. (7) by Poisson's summation formula into an integral which is then asymptotically evaluated for large s and therefore small T by the saddle point method. This has been elegantly carried out for the single-shell case in Ref. [10]. The application of this procedure to the double-shell case is rather complicated and is outside the scope of the present report.

Figure 6 shows the time histories of the radial velocities at various locations of the inner shell. The presence of the outer shell causes little change of the early time values and the profiles of $\dot{w}(\theta,T)$. Other than $\theta = \pi/2$, the values of $\dot{w}(\theta,T)$ at later time are reduced primarily due to the reduction of the $\dot{w}_1$ term previously shown in Figure 3.

The hoop stress resultant $N_\theta(\theta,T)$(force per unit length)at the middle surface of the inner shell is calculated by

$$N_\theta(\theta,T) = \frac{Eh}{1-\nu^2} \quad (w + \frac{\partial v}{\partial \theta}) \qquad (40)$$

and its time histories at $\theta = 0$, $\pi/2$ and π are plotted in Figure 7. Again, the shielding of the outer shell has little effect on the stresses at very early time and does not change the profiles of the stress time histories. At later time, the stresses are reduced due to the reduction of the w_0 term. The present results also clearly demostrates the generation of circumferential stress waves at the inner shell as can be observed from the repeated appearance of peaks in the stress time histories at appropriate time intervals. These stress waves have the same features as those of the single-shell case discussed in Reference [3,4]. They encircle the cylinder with the dilatational wave speed in both directions. Since the attenuation due to radiation is small, many circum-navigation may be observed.

Similar to the double-spherical-shell case [1], it could also be summarized here that the primary effects of a thin outer shell are to reduce the breathing and translational motions of the inner shell and these could be simply calculated from Eqs. (22) and (24).

ACKNOWLEDGMENT

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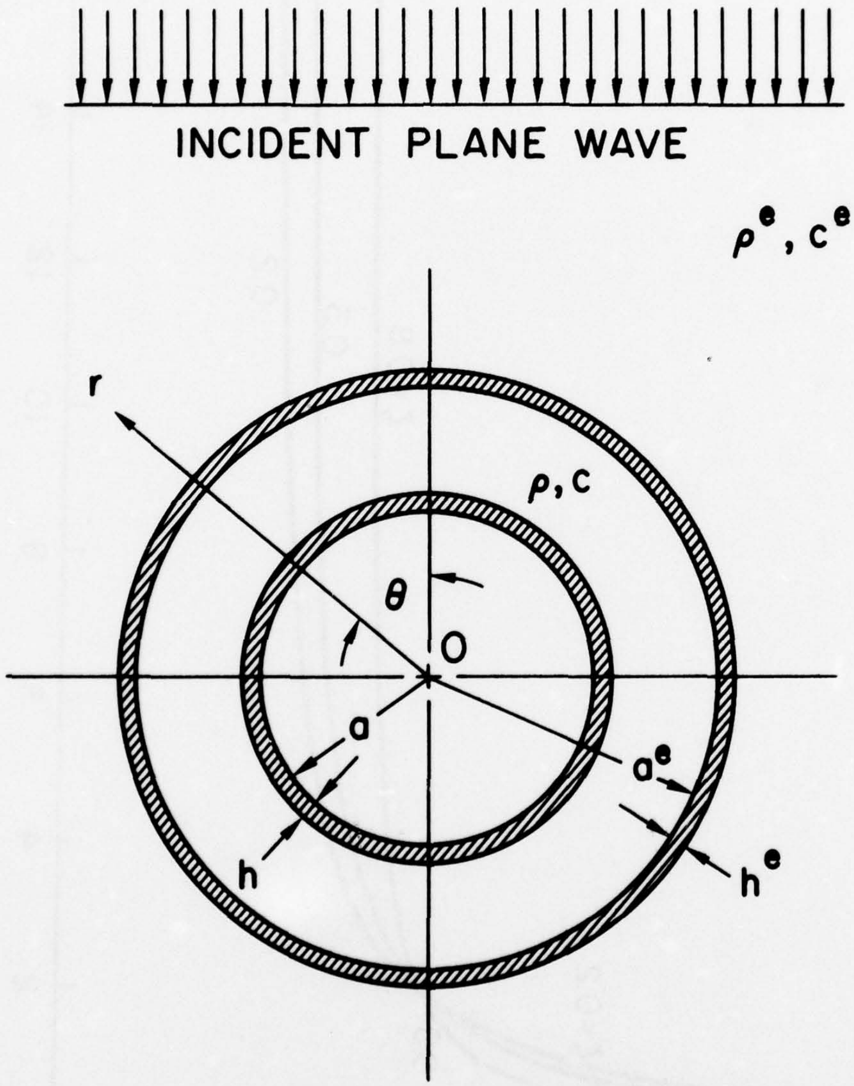

Fig. 1 - Geometry of the problem

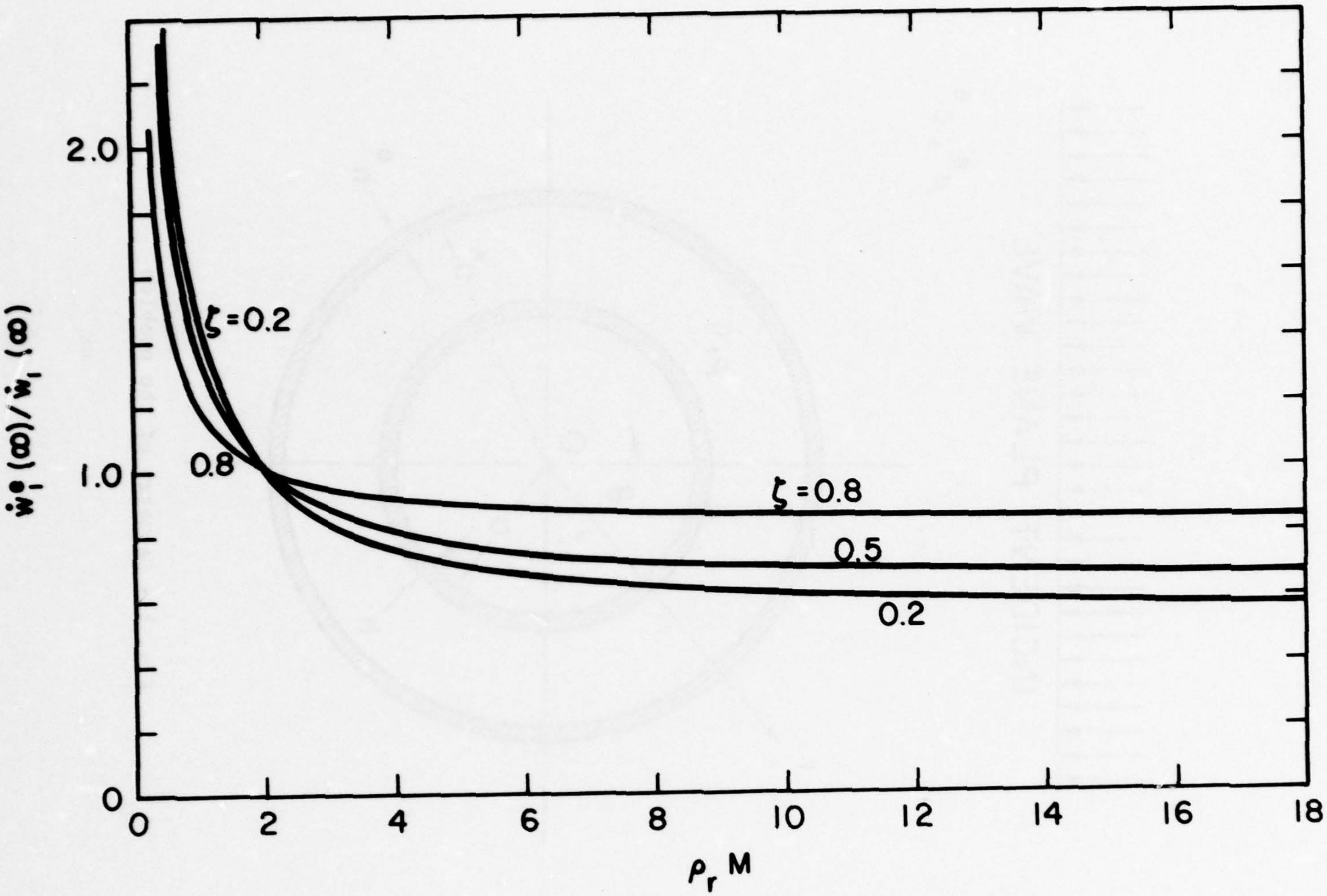

Fig. 2 - Ratios of the late-time translational velocities

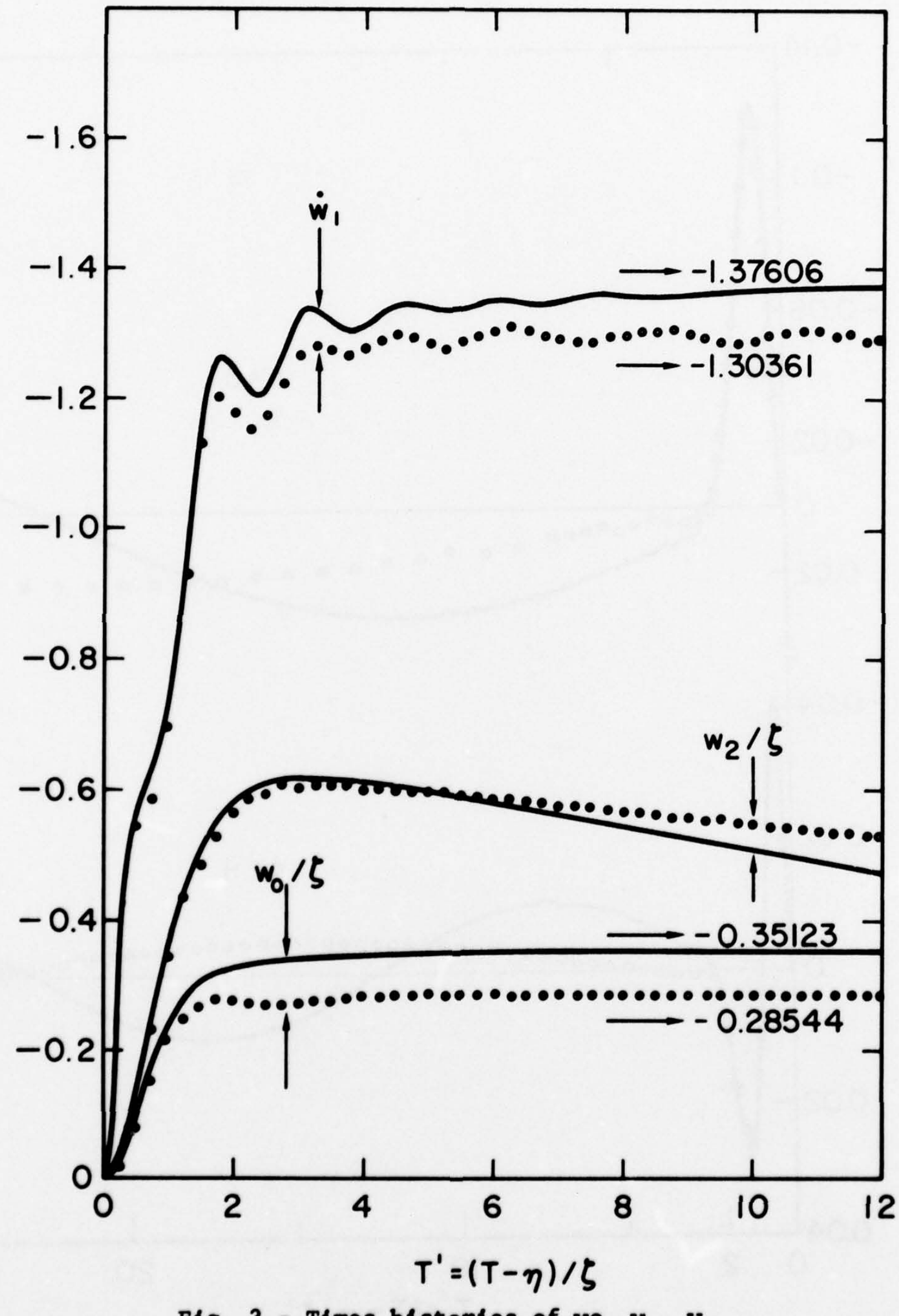

Fig. 3 - Times histories of wo, w_1, w_2

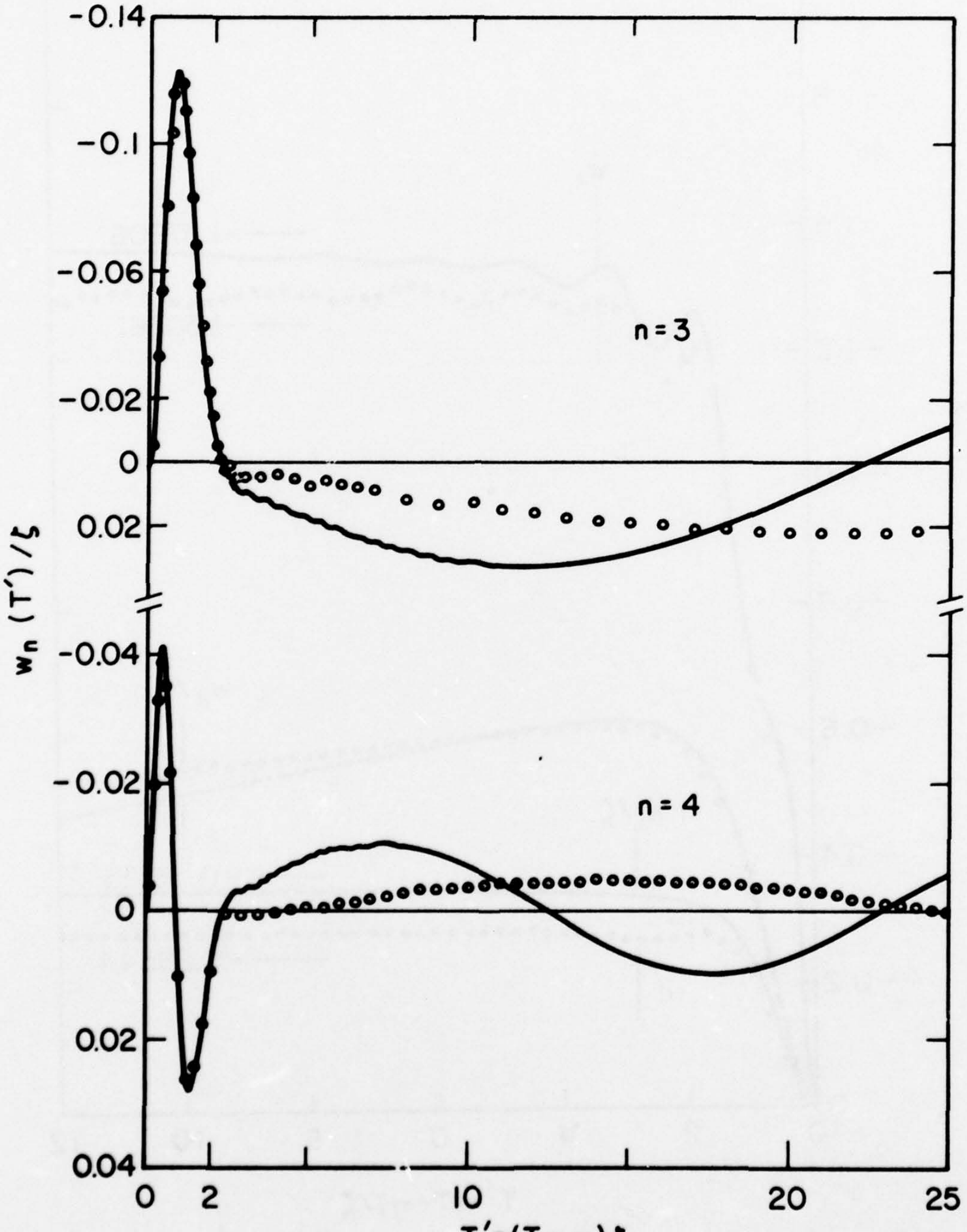

Fig. 4 - Time histories of w_3 and w_4

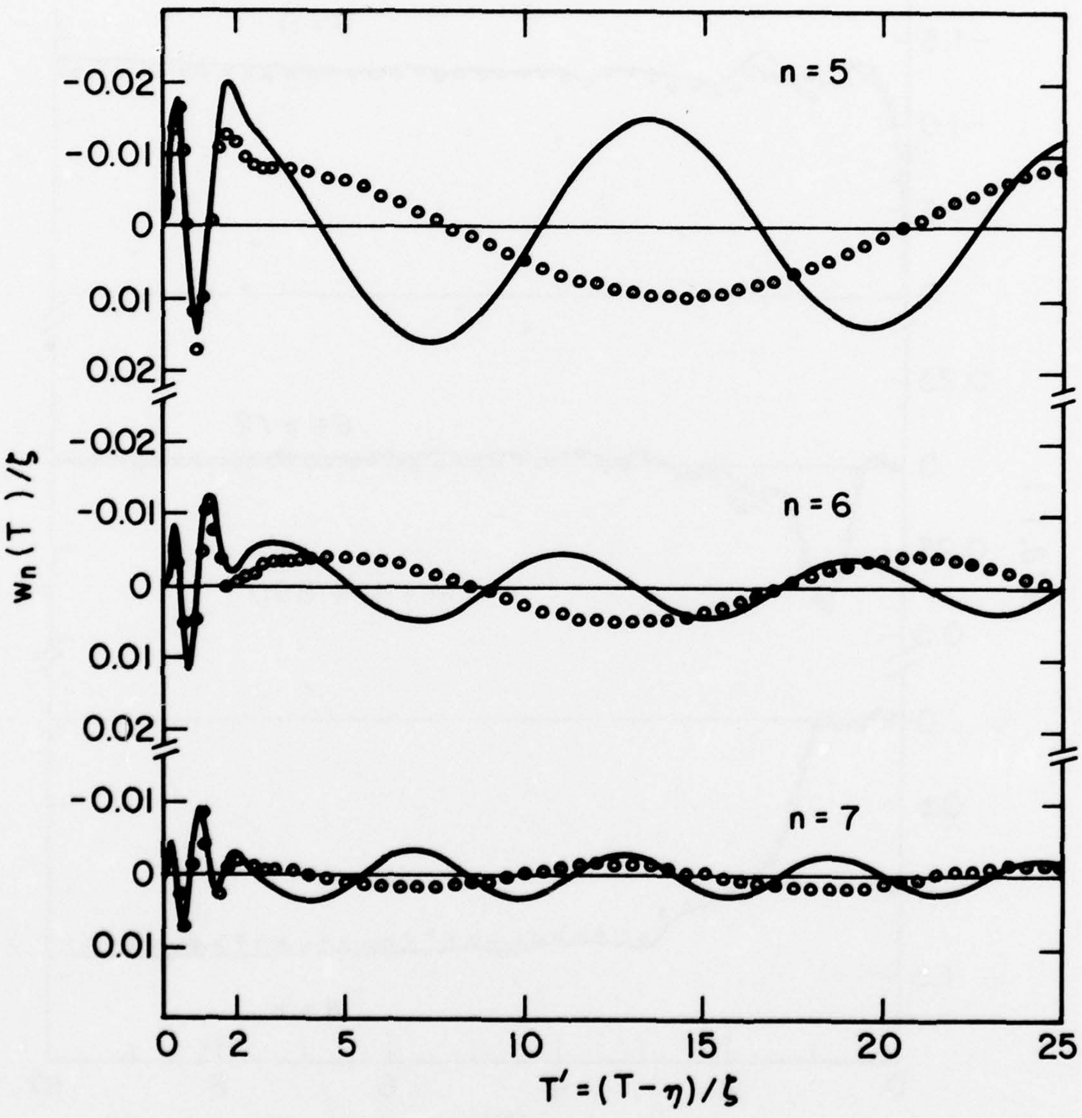

Fig. 5 - Time histories of w_5, w_6 and w_7

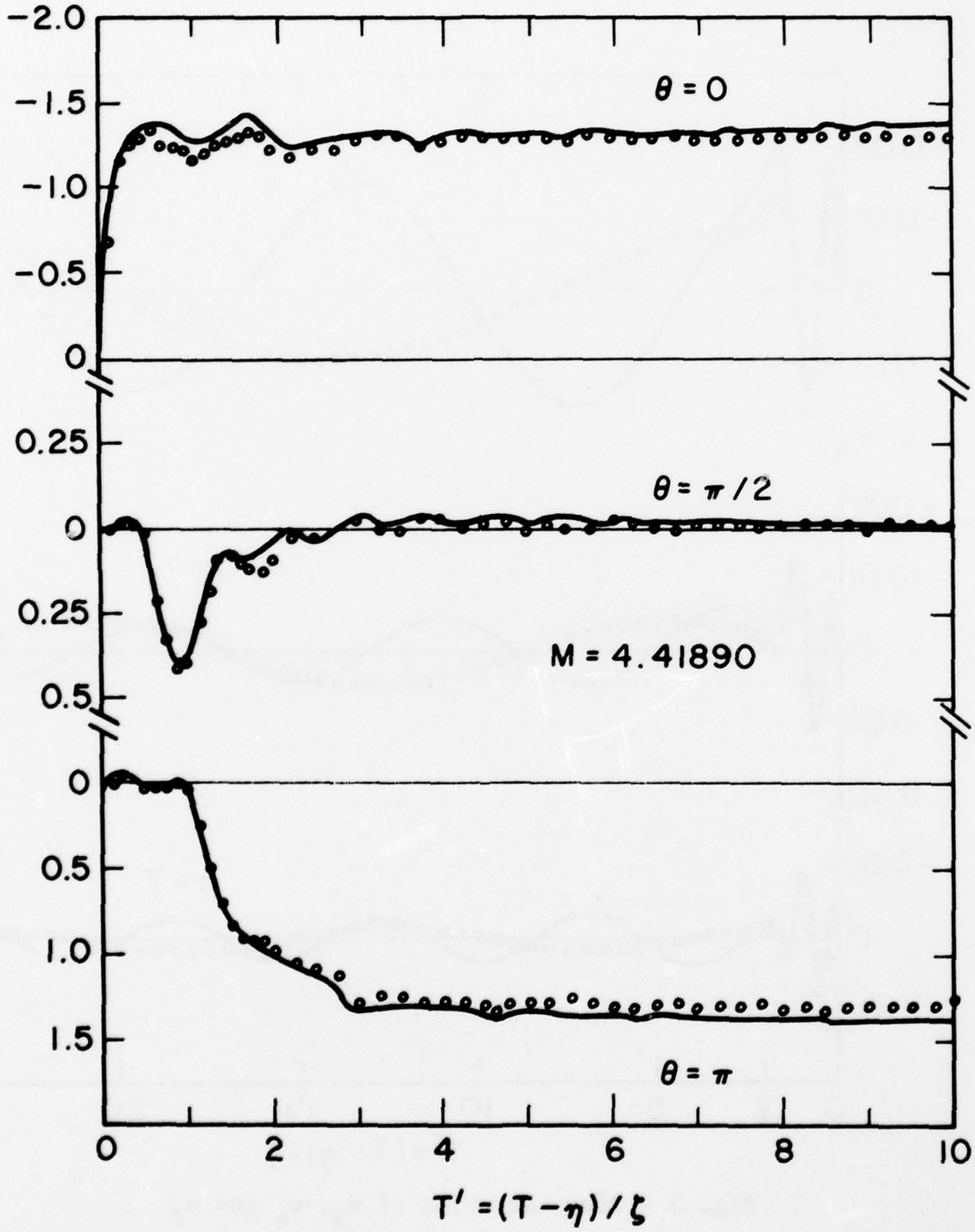

Fig. 6 - Time histories of radial velocities of the inner shell

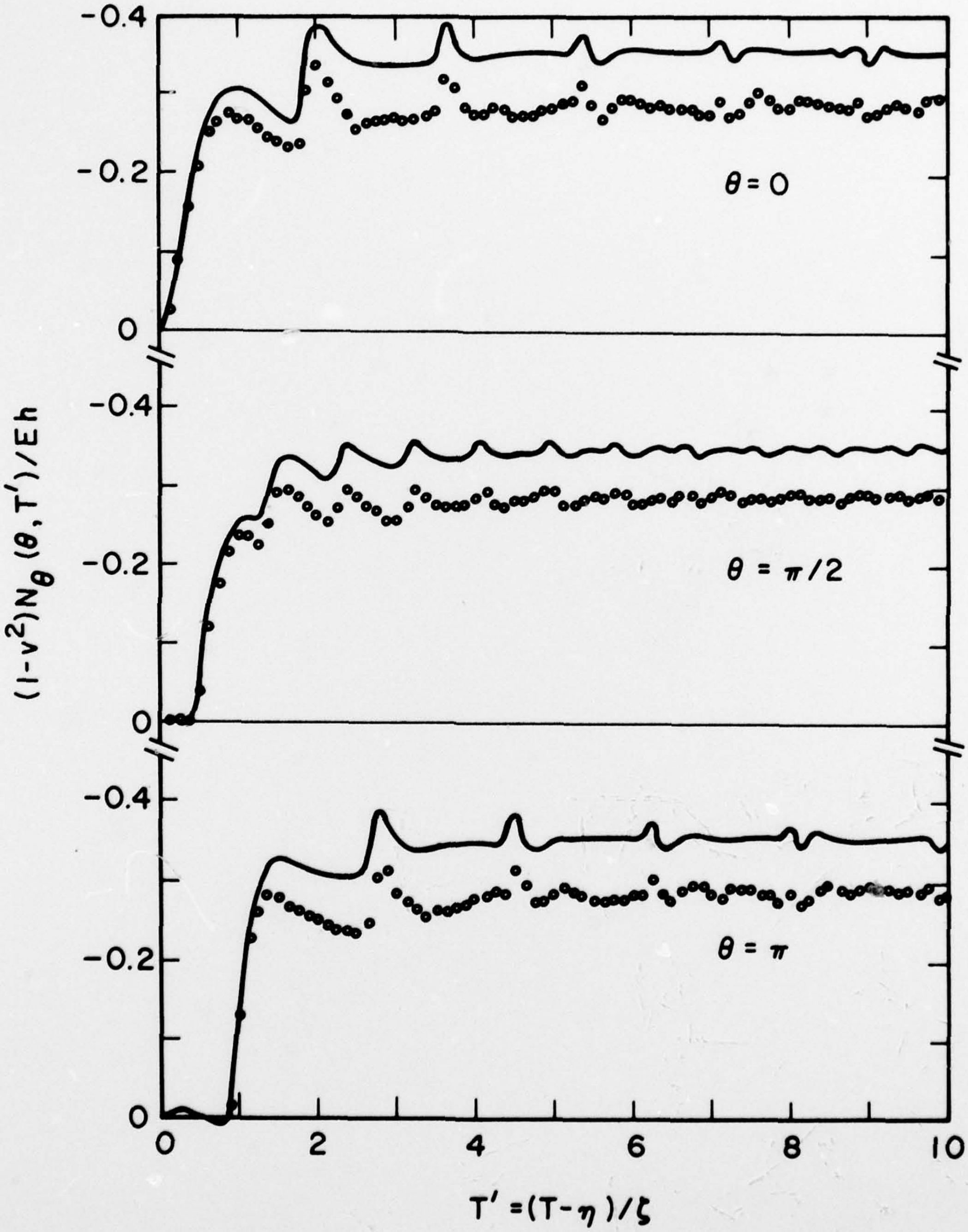

Fig. 7 - Time histories of the hoop stresses at the middle surface of the inner shell

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